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# Ambipolar lateral diffusion of photo-induced carriers in a moderate magnetic field

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**Abstract**
The ambipolar lateral diffusion of photo-induced charge carriers restricted in a plane of a quantum well (QW) under a moderate (non-quantizing) magnetic field is studied theoretically in the framework of the drift-diffusion model. The continuity equation for this case is solved exactly. The analytical expressions for the concentration of photo-induced electrons and the built-in electric field in the practically important cases of uniform and bell-shaped light beams are obtained in a closed form. It is shown that the ambipolar lateral diffusion of photo-induced charge carriers can be suppressed in InGaAs/GaAs QWs by a moderate magnetic field of ∼0.5 mT.

## 1. Introduction

Carrier diffusion plays an important role in the investigation and design of semiconductor optoelectronic devices, especially lasers and light-emitting diodes (LEDs) [1]. In heterostructures the potential profile and carrier transport can be tailored in the growth direction, while in the layers plane transport is determined by diffusion [1]. An imbalance of electron and hole concentrations can produce an electric field which causes a drift term in the total current density. The transport of photo-excited charge carriers in an undoped semiconductor under conditions of charge neutrality and the absence of current includes both diffusion and drift processes and results in ambipolar diffusion [1]. In recent decades, carrier diffusion in InGaAs/GaAs quantum wells (QWs) restricted in a plane has been experimentally investigated using different techniques such as cathodoluminescence, the transient-grating method and the high resolution time-of-flight technique (see [1–6] and references therein). The measured value of the QW ambipolar diffusion length $L_{\mathrm{ambi}}$ is shown to be about 2.7 $\mu$m [1]. Significant in-plane diffusion in high quality InGaAs/GaAs QWs results in deterioration of their

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current–voltage and light–current characteristics [1]. The diffusion length limits the active device diameter in QW vertical-cavity surface emitting lasers (VCSELs) [1] and increases threshold currents in QW laser diodes [5]. The control of carrier diffusion is essential for the fabrication of efficient LEDs and lasers with dimensions in the submicrometre scale, especially for the realization of ultralow threshold lasers and single quantum dot (QD) LEDs to be used as single-photon sources for quantum communications [1]. The proposed methods for suppression of carrier diffusion are all of a technological nature, such as replacement of QWs with QDs and disordered QWs [1].

The theoretical approach to the problem of lateral diffusion is based on the classical drift-diffusion model for electron–hole plasma in semiconductors [7, 8]. The in-plane motion of charge carriers in QWs can be considered classically because the electron energies within QWs consist of the confined-state energy arising from the spatial quantization in the direction normal to the interface and the free-electron energies in the directions parallel to the interface [9]. Previously, the analysis was carried out for the case without external fields [5, 6]. Comparison between these calculations and the experimental data showed a good agreement [5]. The peculiarities of the lateral diffusion of carriers in applied external fields are interesting [2]. Study of the diffusion of optically generated carriers in the presence of a magnetic field should yield new insights into electronic magnetotransport in low-dimensional semiconductors. However, to the best of our knowledge, the influence of the magnetic field on the lateral diffusion of photocarriers in heterostructures has not yet been investigated theoretically.

In this paper we have studied the lateral ambipolar diffusion of photocarriers in a uniform constant magnetic field **B**, taking into account the spatial distribution of the light beam. The theoretical investigation concerns the lateral motion of three-dimensional (3D) electrons and holes in an optically induced inhomogeneity with two-dimensional (2D) symmetry. We have limited our analysis to moderate non-quantizing magnetic fields. The analytical solutions in a closed form have been obtained for both uniform and bell-shaped beams. We evaluated the built-in electric field **E**. We have shown that the lateral diffusion of carriers in InGaAs/GaAs QWs can be confined to a region with a dimension $L_{\mathrm{ambi}} \lesssim 0.5\ \mu$m by using a moderate magnetic field $B \sim 0.5$ mT perpendicular to the plane of the layers.

The paper is constructed as follows. In section 2, the ambipolar diffusion equation in the presence of an optical beam and the magnetic field is derived. The closed-form analytical expressions for the electron concentration and the built-in electric field for the general case and for practically important cases of uniform and bell-shaped light beams are obtained in section 3. Conclusions are presented in section 4. The evaluation of the necessary integrals is presented in the appendix.

## 2. Ambipolar diffusion of the photo-induced charge carriers in a magnetic field

We start with the continuity equations for electron and hole concentrations $n$ and $p$ [7]. They have the form

$$\frac{\partial n}{\partial t} = -\mathrm{div}\,\mathbf{F}_n + G_n - R_n, \qquad \frac{\partial p}{\partial t} = -\mathrm{div}\,\mathbf{F}_p + G_p - R_p \tag{1}$$

where $\mathbf{F}_n = -\mathbf{j}_n/q = \mathbf{v}_n n$, $\mathbf{F}_p = \mathbf{j}_p/q = \mathbf{v}_p p$ are the electron and hole flux densities, $\mathbf{j}_{n,p}$ are the electron and hole current densities, $G_{n,p}$ and $R_{n,p}$ are the electron and hole generation and recombination rates, $\mathbf{v}_{n,p}$ are the electron and hole velocities, respectively, and $q$ is the magnitude of the electron charge. For photo-generation, the generation rate has the form [7, 10]

$$G_n = G_p = (1 - R)\,\alpha \frac{\eta I\,(r)}{h\nu} \exp\left(-\alpha z\right) \tag{2}$$

where $R$ is the reflection coefficient at the surface of the semiconductor layer, $\alpha$ is the material absorption coefficient, $\eta$, $I(r)$, $h$, $\nu$ are the quantum efficiency, the light beam intensity, the Planck constant and the photon frequency, respectively, and coordinates $(r, z)$ are chosen to be in the layer plane and perpendicular to it, respectively. The absorption coefficient $\alpha \sim 10^4$ cm$^{-1}$ for GaAs [11] for the frequencies $\nu$ such that $h\nu \gtrsim E_g$, where $E_g$ is the semiconductor energy gap. Hence, for nano-layers with a thickness $d \ll 1/\alpha$ the dependence of all variables on $z$ in the radial diffusion equation can be neglected [12], and the exponential factor in equation (2) $\exp(-\alpha z) \sim 1$ can be dropped. As a result, the analysis of 3D electron and hole motion reduces to the analysis of their lateral components.

In the framework of the hydrodynamic model, the electron and hole velocities $\mathbf{v}_{n,p}$ are determined by the following equations of motion in an inhomogeneous plasma, in the presence of an external magnetic field $\mathbf{B} = \mathbf{a}_z B$ and a built-in electric field $\mathbf{E} = (\mathbf{a}_x E_x + \mathbf{a}_y E_y) = -\nabla V$, where $\mathbf{a}_{x,y,z}$ are the unit vectors and $V$ is a scalar potential [8]:

$$m^*_{n,p}\left[\frac{\partial \mathbf{v}_{n,p}}{\partial t} + \left(\mathbf{v}_{n,p}\cdot\nabla\right)\mathbf{v}_{n,p}\right] = \mp q\left\{\mathbf{E} + \left[\mathbf{v}_{n,p}\times\mathbf{B}\right]\right\} - \mathbf{f}_{n,p} - m^*_{n,p}\frac{\mathbf{v}_{n,p}}{\tau_{n,p}}. \tag{3}$$

Here $\mathbf{f}_n = n^{-1}\nabla P_n$, $\mathbf{f}_p = p^{-1}\nabla P_p$, and $m^*_{n,p}$, $\tau_{n,p}$, $P_n = k_B T n$, $P_p = k_B T p$ are the electron and hole effective masses, the electron and hole lifetimes, the pressures of electrons and holes, respectively, $T$ is the temperature and $k_B$ is the Boltzmann constant. When the time of radiation $\Delta t_{\mathrm{rad}}$ is sufficiently large compared to the electron and hole recombination times $\Delta t_{\mathrm{rad}} \gg \tau_n, \tau_p$, the situation is assumed to be steady-state $(\partial/\partial t = 0)$. In the first approximation, the nonlinear terms in equation (3) can be ignored. The lifetime dependence on the carrier concentration can be neglected [1, 2, 5]. Under the influence of the transverse magnetic field, the charge carriers start moving in directions perpendicular to their initial trajectories, thus reducing their free path.

Decomposing vector equation (3) we obtain the linear equations for the electron and hole velocity components $v_{nx,y}$ and $v_{px,y}$

$$\frac{k_B T}{m^*_n n}\frac{\partial n}{\partial r_{i,k}} - \frac{q}{m^*_n}v_{nk,i}B + \frac{q}{m^*_n}\frac{\partial V}{\partial r_{i,k}} - \frac{1}{\tau_n}v_{ni,k} = 0 \tag{4}$$

$$\frac{k_B T}{m^*_p p}\frac{\partial p}{\partial r_{i,k}} + \frac{q}{m^*_p}v_{pk,i}B - \frac{q}{m^*_p}\frac{\partial V}{\partial r_{i,k}} - \frac{1}{\tau_p}v_{pi,k} = 0 \tag{5}$$

where $r_i = x$, $r_k = y$ and the indices $i, k$ stand for $x, y$. Substituting the solutions of equations (4) and (5) $v_{nx,y}$ and $v_{px,y}$ into equation (1) in the steady-state case, assuming radial symmetry of both the photocarrier concentrations $n(r)$, $p(r)$ and the electric field potential $V(r)$ due to the radial symmetry of the radiation beam (2) and transforming the Cartesian coordinates $(x, y)$ to the cylindrical ones we obtain the following equation:

$$-D_{n,p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_{n,p}}{\partial r}\right) \pm \mu_{n,p}\frac{1}{r}\frac{\partial}{\partial r}\left(rC_{n,p}\frac{\partial V}{\partial r}\right) = \left(G_n - R_{n,p}\right)\left(1 + \omega^2_{cn,p}\tau^2_{n,p}\right) \tag{6}$$

where $\omega_{cn,p} = qB/m^*_{n,p}$, $\mu_{n,p}$ and $D_{n,p} = \mu_{n,p}k_B T/q$ are the electron and hole cyclotron frequencies, mobilities and diffusion coefficients, respectively, $C_n \equiv n$, $C_p \equiv p$ and the following identities for the radially symmetric case are taken into account:

$$\frac{\partial V(r)}{\partial y}\frac{\partial C_{n,p}(r)}{\partial x} - \frac{\partial V(r)}{\partial x}\frac{\partial C_{n,p}(r)}{\partial y} = 0. \tag{7}$$

In the typical case of photocarrier generation the following conditions are valid [10]:

$$n = n_0 + n_{\mathrm{ph}}, \qquad p = p_0 + p_{\mathrm{ph}}, \qquad n_{\mathrm{ph}} = p_{\mathrm{ph}} \tag{8}$$

$$n_{\mathrm{ph}} \ll n_0 = \mathrm{const}, \qquad p_{\mathrm{ph}} \ll p_0 = \mathrm{const} \tag{9}$$

and $R_{n,p} = n_{\text{ph}}/\tau_{n,p}$, where $n_0$, $p_0$ are the equilibrium carrier concentrations. In the particular case of n-type GaAs, the equilibrium hole concentration $p_0 \ll n_0$, and only the first inequality of conditions (9) is essential. In the case of lateral diffusion it is helpful to replace the carrier concentration per unit volume with the concentration per unit area, or sheet carrier density [5] $n_{\text{ph2D}}$, as follows: $n_{\text{ph2D}} = dn_{\text{ph}}$. Then, substituting conditions (8), (9) and $n_{\text{ph2D}}$ into equation (6), multiplying these equations by $\mu_p$ and $\mu_n$ and adding them we obtain the ambipolar lateral diffusion equation for electron gas in the magnetic field:

$$\frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial n_{\text{ph2D}}}{\partial u}\right) - n_{\text{ph2D}} = -N_{\text{2D}}(u) \tag{10}$$

where the dimensionless variable $u = r/L_{\text{ambi}}$, the ambipolar diffusion length $L_{\text{ambi}}$ and the photon surface density $N_{\text{2D}}(u)$ are given by

$$L_{\text{ambi}}^2 = \frac{(n_0 + p_0) L_{\text{peff}}^2 L_{\text{neff}}^2}{\left(p_0 L_{\text{peff}}^2 + n_0 L_{\text{neff}}^2\right)}; \qquad N_{\text{2D}}(u) = (1 - R)\,\alpha d\frac{\eta I(u)\langle\tau\rangle}{h\nu}. \tag{11}$$

Here $L_{\text{neff}}$, $L_{\text{peff}}$ and $\langle\tau\rangle$ are the effective electron and hole diffusion lengths in the magnetic field and the effective photocarrier lifetime given by

$$L_{n,\text{peff}}^2 = \frac{D_{n,p}\tau_{n,p}}{\left(1 + \omega_{cn,p}^2\tau_{n,p}^2\right)}; \qquad \langle\tau\rangle = \frac{\tau_p n_0 L_{\text{neff}}^2 + \tau_n p_0 L_{\text{peff}}^2}{p_0 L_{\text{peff}}^2 + n_0 L_{\text{neff}}^2}. \tag{12}$$

For the n-type material with $n_0 \gg p_0$, $L_{\text{ambi}} \approx L_{\text{peff}}$, and in the opposite case $L_{\text{ambi}} \approx L_{\text{neff}}$. For $\tau_n \sim \tau_p = \tau$ and $n_0 = p_0$, expressions (11) and (12) yield for $L_{\text{ambi}}$ and $\langle\tau\rangle$ in the limiting case $\omega_{cn,p}^2\tau_{n,p}^2 \gg 1$

$$L_{\text{ambi}} \approx \sqrt{\frac{2D_n D_p}{\tau\left(D_n\omega_{cp}^2 + D_p\omega_{cn}^2\right)}}; \qquad \langle\tau\rangle \approx \tau. \tag{13}$$

The dependence of $L_{\text{ambi}}$ on the magnetic field $B$ is presented in figure 1 for typical values of material parameters for GaAs [1, 5, 11] $n_0 = 2 \times 10^{18}$ cm$^{-3}$, $\tau_n = \tau_p = 3 \times 10^{-9}$ s, $\mu_n = 8000$ cm$^2$ V$^{-1}$ s$^{-1}$, $\mu_p = 400$ cm$^2$ V$^{-1}$ s$^{-1}$, $m_n^* = 0.067m_0$, $m_p^* = 0.45m_0$ where $m_0$ is the free-electron mass. It is seen that $L_{\text{ambi}} \sim 0.5$ $\mu$m can be reached at a moderate magnetic field $B \sim 0.5$ mT.

## 3. Evaluation of the photocarrier concentration and the built-in electric field

### *3.1. General solution of the ambipolar diffusion equation*

Inhomogeneous equation (10) can be solved by using the Hankel transformation given for a function $f(u)$ by [13]

$$\tilde{f}(\xi) = \int_0^\infty u f(u) J_0(\xi u)\,\mathrm{d}u; \qquad f(u) = \int_0^\infty \xi \tilde{f}(\xi) J_0(\xi u)\,\mathrm{d}\xi \tag{14}$$

where $J_0(x)$ is a Bessel function of the first kind of zeroth order [14]. Integrating the left-hand side of equation (10) by parts and using the properties of the Bessel function we obtain

$$\tilde{n}_{\text{ph2D}}(\xi) = \frac{\tilde{N}_{\text{2D}}(\xi)}{(\xi^2 + 1)} \tag{15}$$

where $\tilde{n}_{\text{ph2D}}(\xi)$ and $\tilde{N}_{\text{2D}}(\xi)$ are the Hankel transforms (14) of the quantities $n_{\text{ph2D}}(u)$ and $N_{\text{2D}}(u)$. The solution of equation (10) can be obtained by using the inverse Hankel transform defined by the second expression of (14):

$$n_{\text{ph2D}}(u) = \int_0^\infty \xi\frac{\tilde{N}_{\text{2D}}(\xi)}{(\xi^2 + 1)} J_0(\xi u)\,\mathrm{d}\xi. \tag{16}$$

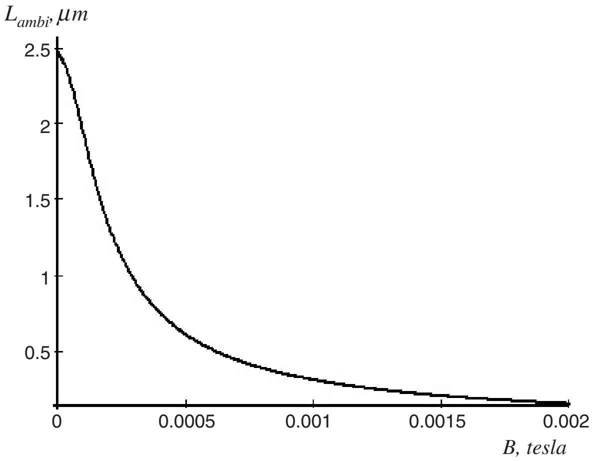

**Figure 1.** The dependence of the ambipolar diffusion length $L_{\mathrm{ambi}}$ on the magnetic field $B$.

Substituting the integral representation of $\widetilde{N}_{2\mathrm{D}}(\xi)$ (14) into expression (16) and changing the order of integration we obtain

$$n_{\mathrm{ph2D}}(u) = \int_0^{\infty} u_1 N_{2\mathrm{D}}(u_1)\, \mathrm{d}u_1 \Phi(u, u_1) \tag{17}$$

where the internal integral $\Phi(u, u_1)$ in expression (17) is known [15]

$$\Phi(u, u_1) = \int_0^{\infty} \frac{\xi}{\left(\xi^2 + 1\right)} J_0(\xi u)\, J_0(\xi u_1)\, \mathrm{d}\xi = \begin{cases} I_0(u) K_0(u_1), & u_1 > u \\ I_0(u_1) K_0(u), & u_1 < u. \end{cases} \tag{18}$$

Here $I_\nu(u)$ and $K_\nu(u)$ are the modified Bessel functions of order $\nu$ [14, 15].

Taking into account relationships (9)–(12) and eliminating $n_{\mathrm{ph2D}}$ from the continuity equation (6) we obtain the following expression for the radial component of the built-in electric field $E_r = -\partial V / \partial r$.

$$E_r = E_{r0} \left[ \frac{A_1}{u} \int_0^u u_1 N_{2\mathrm{D}}(u_1)\, \mathrm{d}u_1 + A_2 \frac{\partial n_{\mathrm{ph2D}}}{\partial u} \right] \tag{19}$$

where $E_{r0} = k_{\mathrm{B}} T / q d$ and

$$A_1 = \frac{L_{\mathrm{ambi}} \left(\tau_p - \tau_n\right)}{\left(\tau_p n_0 L_{\mathrm{neff}}^2 + \tau_n p_0 L_{\mathrm{peff}}^2\right)}; \qquad A_2 = \frac{\left(L_{\mathrm{peff}}^2 - L_{\mathrm{neff}}^2\right)}{\left(L_{\mathrm{neff}}^2 n_0 + L_{\mathrm{peff}}^2 p_0\right) L_{\mathrm{ambi}}}. \tag{20}$$

### *3.2. The uniform light beam*

Consider first the case of a uniform light beam given by

$$N_{2\mathrm{D}}(u) = \begin{cases} N_0, & u \leqslant u_0 \\ 0, & u > u_0 \end{cases} \tag{21}$$

where $u_0 = r_0 / L_{\mathrm{ambi}}$, and $r_0 \sim$ (5–10) $\mu$m is the radius of the light beam. For a photon flux density per unit area [10] $10^{18}$ photon cm$^{-2}$ s$^{-1}$, $\langle\tau\rangle \sim 10^{-9}$ s and $N_0 \sim 10^9$ cm$^{-2}$, condition (9) is valid for a sheet carrier density [1, 2] $n_{02D} \sim (10^{11}\text{–}10^{12})$ cm$^{-2}$. Substituting expression (21) into expressions (17) and (19) we obtain

$$n_{\mathrm{ph2D}}(u) = N_0 \int_0^{u_0} u_1 \,\mathrm{d}u_1 \Phi(u, u_1) \tag{22}$$

$$E_r(u) = E_{r0} N_0 \times \begin{cases} \left[ \dfrac{A_1 u}{2} + A_2 \dfrac{\partial}{\partial u} \left( \displaystyle\int_0^{u_0} u_1 \,\mathrm{d}u_1 \Phi(u, u_1) \right) \right], & u \leqslant u_0 \\ \left[ \dfrac{A_1 u_0^2}{2u} + A_2 \dfrac{\partial}{\partial u} \left( \displaystyle\int_0^{u_0} u_1 \,\mathrm{d}u_1 \Phi(u, u_1) \right) \right], & u > u_0. \end{cases} \tag{23}$$

The first term in expressions (27) is due to the space charge distribution inside and outside the beam region and is similar to the field of a dielectric cylinder filled with the space charge. It vanishes in the particular case when $\tau_p - \tau_n = 0$. The second term in expressions (27) is caused by the inhomogeneity of the photocarrier concentration.

Consider first the solution inside the beam region $u \leqslant u_0$. Substituting results (18) into expression (22) we obtain

$$n_{\mathrm{ph2D}}(u) = N_0 \left[ K_0(u) \int_0^u u_1 \,\mathrm{d}u_1 I_0(u_1) + I_0(u) \int_u^{u_0} u_1 \,\mathrm{d}u_1 K_0(u_1) \right]. \tag{24}$$

The integrals in expression (24) are easily evaluated by virtue of the identities [15]

$$z \frac{\mathrm{d}}{\mathrm{d}z} K_\nu(z) + \nu K_\nu(z) = -z K_{\nu-1}(z); \qquad z \frac{\mathrm{d}}{\mathrm{d}z} I_\nu(z) + \nu I_\nu(z) = z I_{\nu-1}(z). \tag{25}$$

The result is given by

$$n_{\mathrm{ph2D}}(u) = N_0 \left[1 - u_0 K_1(u_0) I_0(u)\right]. \tag{26}$$

Substituting expression (26) into the first of formulae (23) we obtain for the built-in electric field in the region $u \leqslant u_0$

$$E_r(u) = E_{r0} N_0 \left[ \frac{A_1 u}{2} - A_2 u_0 K_1(u_0) I_1(u) \right]. \tag{27}$$

Outside the beam region for $u > u_0$, we obtain for the electron concentration, using the second expression of (18),

$$n_{\mathrm{ph2D}}(u) = N_0 K_0(u) \int_0^{u_0} u_1 \,\mathrm{d}u_1 I_0(u_1) = N_0 u_0 I_1(u_0) K_0(u). \tag{28}$$

Substituting expression (28) into the second of formulae (23) we get

$$E_r(u) = E_{r0} N_0 u_0 \left[ \frac{A_1 u_0}{2u} - A_2 I_1(u_0) K_1(u) \right]. \tag{29}$$

Using the asymptotic expression of the modified Bessel functions [14, 15] $K_\nu(u) \sim \sqrt{\pi/(2u)}\exp(-u)$ we obtain the following expressions for the electron concentration $n_{\mathrm{ph2D}}(u)$ and the built-in electric field $E_r(u)$ at large distances $u \gg u_0 > 1$ from the beam:

$$n_{\mathrm{ph2D}}(u) \approx N_0 u_0 I_1(u_0) \sqrt{\frac{\pi}{2u}} \exp(-u) \tag{30}$$

$$E_r(u) \approx E_{r0} N_0 u_0 \left[ \frac{A_1 u_0}{2u} - A_2 I_1(u_0) \sqrt{\frac{\pi}{2u}} \exp(-u) \right]. \tag{31}$$

The normalized electron concentration $n_{\mathrm{ph2D}}(u)/N_0$ for a uniform beam is shown in figure 2. For typical values of $L_{\mathrm{ambi}}$, the parameter $u_0 \gg 1$. The profile of the electron concentration is determined by the radius of the beam. The built-in electric field $E_r(u)$ for $B = 5 \times 10^{-4}$ T and $B = 10^{-3}$ T, $\tau_n \sim \tau_p \sim 10^{-9}$ s, $|\tau_p - \tau_n| \sim 10^{-9}$ s and $u_0 = 10$ is shown in figure 3. The

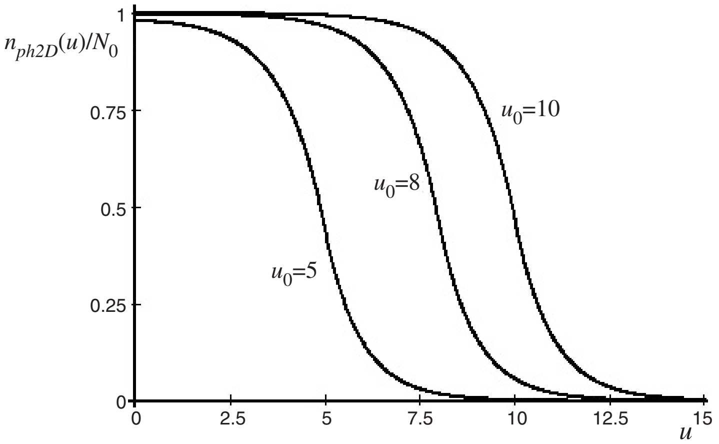

**Figure 2.** The normalized 2D concentration of electrons $n_{\mathrm{ph2D}}/N_0$ for the uniform light beams with $u_0 = 5; 8; 10$.

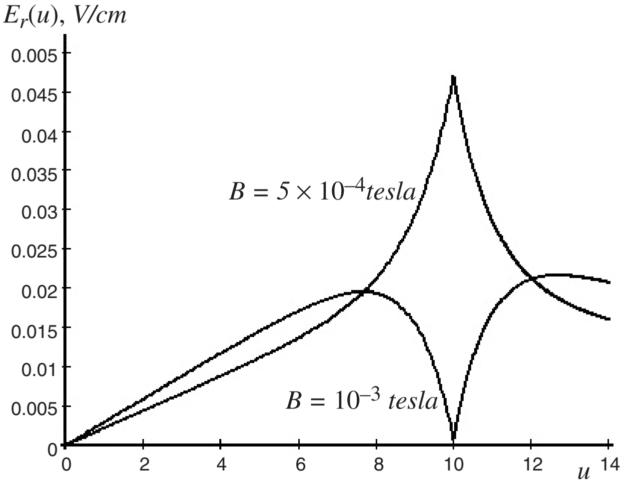

**Figure 3.** The built-in electric field $E_r(u)$ for $B = 5 \times 10^{-4}$ T; $10^{-3}$ T, $\tau_n - \tau_p \neq 0$, $u_0 = 10$.

coefficient $A_2$ in expressions (27) and (29) can change sign with the increase in the magnetic field because $\omega_{cp} \ll \omega_{cn}$, which results in the suppression of the electric field in the region of the beam as can be seen from figure 3. In the particular case $\tau_p - \tau_n = 0$ the coefficient $A_1$ vanishes. The remaining electric field is shown in figure 4.

### 3.3. A bell-shaped light beam

Consider now a bell-shaped light beam which can be approximated with the modified Bessel function $K_1(u)$

$$N_{\mathrm{2D}}(u) = N_0 u \gamma K_1(\gamma u) \tag{32}$$

where $\gamma$ is a dimensionless fitting parameter. The normalized distributions $N_{\mathrm{2D}}(u)/N_0$ for different values of $\gamma$ are shown in figure 5. Substituting expression (32) into expression (14)

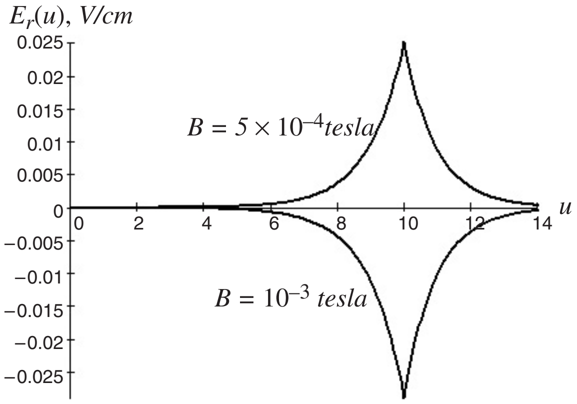

**Figure 4.** The built-in electric field $E_r(u)$ for $B = 5 \times 10^{-4}$ T; $10^{-3}$ T, $\tau_n - \tau_p = 0$, $u_0 = 10$.

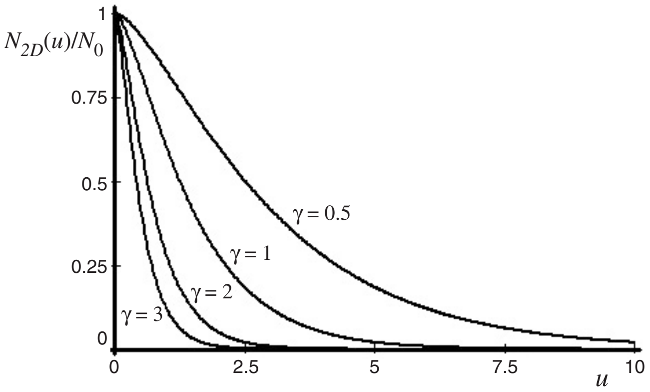

**Figure 5.** The normalized profile of the bell-shaped light beam $N_{2D}/N_0$ for $\gamma = 0.5; 1; 2; 3$.

we obtain the following expression

$$\widetilde{N}_{2D}(\xi) = \gamma N_0 \int_0^\infty u^2 K_1(\gamma u) J_0(\xi u)\,du. \tag{33}$$

The integral in (33) is given by [15]

$$\int_0^\infty u^2 K_1(\gamma u) J_0(\xi u)\,du = \frac{2\gamma^2}{\left(\xi^2 + \gamma^2\right)^2}. \tag{34}$$

Substituting the results (33) and (34) into expression (16) we obtain

$$n_{ph2D}(u) = 2\gamma^2 N_0 \int_0^\infty \frac{\xi}{\left(\xi^2 + 1\right)\left(\xi^2 + \gamma^2\right)^2} J_0(\xi u)\,d\xi. \tag{35}$$

Evaluation of the integral in expression (35) is presented in the appendix. It yields

$$n_{ph2D}(u) = \begin{cases} \dfrac{2\gamma^2 N_0}{(1-\gamma^2)} \left\{ \dfrac{\left[K_0(u) - K_0(\gamma u)\right]}{(1-\gamma^2)} + \dfrac{u K_1(\gamma u)}{2\gamma} \right\}, & \gamma \neq 1 \\ N_0 \left(\dfrac{u}{2}\right)^2 K_2(u), & \gamma = 1. \end{cases} \tag{36}$$

We evaluate the built-in electric field $E_r$ by substituting expressions (32) and (36) into expression (19). We obtain the following results for the case when $\gamma \neq 1$:

$$\begin{aligned} E_r(u) = E_{r0}N_0 \bigg\{ & \frac{A_1}{u}\int_0^u u_1^2 \gamma K_1(\gamma u_1)\,\mathrm{d}u_1 + A_2 \frac{2\gamma^2}{(1-\gamma^2)} \\ & \times \frac{\partial}{\partial u}\left(\frac{[K_0(u)-K_0(\gamma u)]}{(1-\gamma^2)} + \frac{uK_1(\gamma u)}{2\gamma}\right)\bigg\}. \end{aligned} \tag{37}$$

Expression (37) can be easily evaluated by using identity (25). The resulting expression takes the form

$$\begin{aligned} E_r(u) = E_{r0}N_0 \bigg\{ & \frac{A_1}{u\gamma^2}\left[2-(\gamma u)^2 K_2(\gamma u)\right] \\ & + \frac{2A_2\gamma^2}{\left(1-\gamma^2\right)^2}\left[-K_1(u)+\gamma K_1(\gamma u)\right] - \frac{A_2\gamma^2}{\left(1-\gamma^2\right)} u K_0(\gamma u)\bigg\}. \end{aligned} \tag{38}$$

It should be noted that $E_r(0) = 0$, since for $u \to 0$ the first term in (38) vanishes: $\lim_{z\to 0}[(2-z^2K_2(z))/z] = 0$ [15]. At large distances from the beam $u \gg u_0 > 1$ we obtain for $\gamma \neq 1$, using the asymptotic expression [14, 15] for $K_\nu(u)$,

$$n_{\mathrm{ph2D}}(u) \approx \frac{\sqrt{2\pi}\gamma^2 N_0}{\left(1-\gamma^2\right)}\left\{\frac{\exp(-u) - \left(\sqrt{\gamma}\right)^{-1}\exp(-\gamma u)}{\left(1-\gamma^2\right)\sqrt{u}} + \frac{\sqrt{u}\exp(-\gamma u)}{2\gamma^{3/2}}\right\} \tag{39}$$

$$\begin{aligned} E_r(u) \approx E_{r0}N_0 \bigg\{ & \frac{A_1}{\gamma^2 u}\left[2-(\gamma u)^{3/2}\sqrt{\frac{\pi}{2}}\exp(-\gamma u)\right] \\ & + \frac{A_2\gamma^2\sqrt{2\pi}\left[\sqrt{\gamma}\exp(-\gamma u) - \exp(-u)\right]}{\left(1-\gamma^2\right)^2\sqrt{u}} \\ & - \frac{A_2\gamma^{3/2}}{\left(1-\gamma^2\right)}\sqrt{\frac{\pi u}{2}}\exp(-\gamma u)\bigg\}. \end{aligned} \tag{40}$$

In the case when $\gamma = 1$, the built-in electric field $E_r$ is given by

$$E_r(u) = E_{r0}N_0\left\{\frac{A_1}{u}\left[2-u^2K_2(u)\right] - A_2\frac{u^2}{4}K_1(u)\right\}. \tag{41}$$

The asymptotic expressions for $n_{\mathrm{ph2D}}$ and $E_r$ for $\gamma = 1$ are given by

$$n_{\mathrm{ph2D}}(u) \approx \frac{N_0 u^{3/2}}{4}\sqrt{\frac{\pi}{2}}\exp(-u) \tag{42}$$

$$E_r(u) \approx E_{r0}N_0\left\{\frac{A_1}{u}\left(2-u^{3/2}\sqrt{\frac{\pi}{2}}\exp(-u)\right) - \frac{A_2u^{3/2}}{4}\sqrt{\frac{\pi}{2}}\exp(-u)\right\}. \tag{43}$$

The normalized electron concentration $n_{\mathrm{ph2D}}/N_0$ and the built-in electric field $E_r(u)$ for the bell-shaped beams with $\gamma = 0.5; 1; 2; 3$ for $B = 5 \times 10^{-4}$ T and $\tau_p - \tau_n \neq 0$ are presented in figures 6 and 7, respectively. It is seen that in this case the electrons are mainly concentrated in the region $r < L_{\mathrm{ambi}}$. The built-in electric field is also rapidly decreasing for $r > L_{\mathrm{ambi}}$. However, it is larger than in the case of a uniform beam. The built-in electric field $E_r(u)$ in the particular case when $A_1 = 0$ is slightly lower and decreases with the distance much more rapidly then in the general case determined by expressions (38), (40) and (41) because the slowly varying term $\sim u^{-1}$ vanishes.

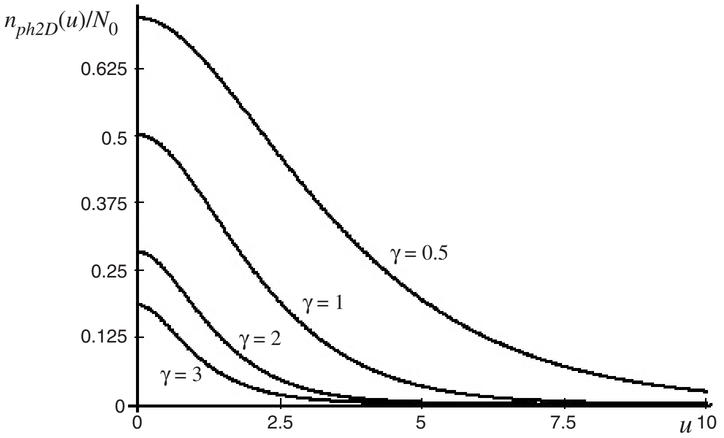

**Figure 6.** The normalized 2D concentration of electrons $n_{\mathrm{ph2D}}/N_0$ for the bell-shaped light beams with $\gamma = 0.5; 1; 2; 3$.

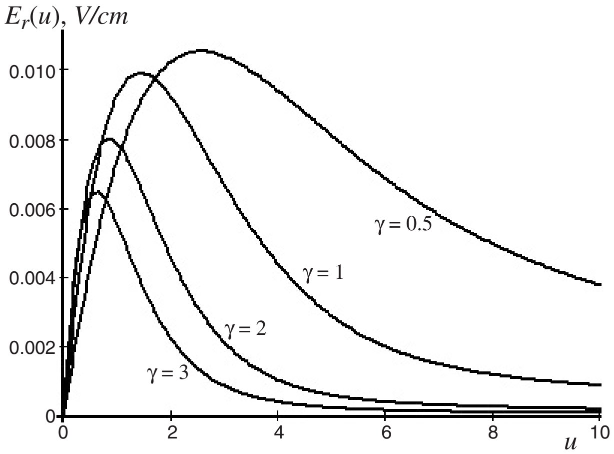

**Figure 7.** The built-in electric field $E_r(u)$ for the bell-shaped light beams with $\gamma = 0.5; 1; 2; 3$. $B = 5 \times 10^{-4}$ T, $\tau_n - \tau_p \neq 0$.

## 4. Conclusions

For the first time we have investigated theoretically the lateral diffusion of photo-induced carriers in a nano-layer subjected to a uniform transverse magnetic field. The ambipolar diffusion equation for the photo-induced carriers in the magnetic field has been solved analytically and the expressions for the distribution of the electron concentration and the built-in electric field have been obtained in a closed form for the two practically important cases: for uniform and bell-shaped light beams. It is shown that lateral diffusion of the carriers can be suppressed by a moderate magnetic field perpendicular to the plane of the layers. The ambipolar diffusion length may be decreased to 0.5 $\mu$m by using $B \sim 0.5$ mT.

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## Appendix. Evaluation of the integral

We wish to evaluate the following integral:

$$i = \int_0^\infty \frac{\xi}{(\xi^2+1)\left(\xi^2+\gamma^2\right)^2} J_0(\xi u)\,\mathrm{d}\xi. \tag{A.1}$$

For $\gamma \neq 1$, the factor $\xi(\xi^2+1)^{-1}(\xi^2+\gamma^2)^{-2}$ in the integrand can be decomposed into simple fractions as follows:

$$\frac{1}{(\xi^2+1)\left(\xi^2+\gamma^2\right)^2} = \frac{1}{\left(1-\gamma^2\right)^2}\left[\frac{1}{\left(\xi^2+1\right)} - \frac{1}{\left(\xi^2+\gamma^2\right)}\right] + \frac{1}{\left(1-\gamma^2\right)\left(\xi^2+\gamma^2\right)^2}. \tag{A.2}$$

Substituting expression (A.2) into (A.1) we obtain

$$i = \frac{1}{\left(1-\gamma^2\right)^2}\int_0^\infty \frac{J_0(\xi u)\xi}{(\xi^2+1)}\,\mathrm{d}\xi - \frac{1}{\left(1-\gamma^2\right)^2}\int_0^\infty \frac{J_0(\xi u)\xi}{\left(\xi^2+\gamma^2\right)}\,\mathrm{d}\xi + \frac{1}{\left(1-\gamma^2\right)}\int_0^\infty \frac{J_0(\xi u)\xi}{\left(\xi^2+\gamma^2\right)^2}\,\mathrm{d}\xi. \tag{A.3}$$

Each of the three integrals in (A.3) reduces to the standard integral of the type [15]

$$\int_0^\infty \frac{J_\nu(bx)\,x^{\nu+1}}{\left(x^2+a^2\right)^{\mu+1}}\,\mathrm{d}x = \frac{a^{\nu-\mu}b^\mu}{2^\mu\Gamma(\mu+1)} K_{\nu-\mu}(ab) \tag{A.4}$$

where $\Gamma(\mu+1)$ is the gamma function [15]. Substituting (A.4) into (A.3) and taking into account the identity $K_{-\nu}(z) = K_\nu(z)$ [14, 15] we obtain

$$i = \frac{\left[K_0(u) - K_0(\gamma u)\right]}{\left(1-\gamma^2\right)^2} + \frac{uK_1(\gamma u)}{2\gamma\left(1-\gamma^2\right)}. \tag{A.5}$$

In the case when $\gamma = 1$ integral (A.1) reduces to the standard form (A.4) and can be written immediately as

$$\int_0^\infty \frac{\xi}{(\xi^2+1)^3} J_0(\xi u)\,\mathrm{d}\xi = \frac{u^2K_2(u)}{8} \tag{A.6}$$

where $a = 1$, $b = u$, $\mu = 2$ and $\nu = 0$.

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